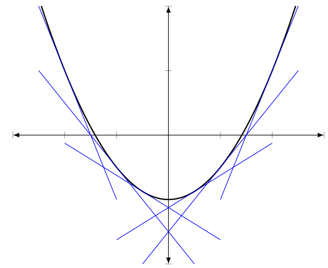
California State University SAN MARCOS

CALCULUS — DERIVATIVE RULES

DEFINITION OF THE DERIVATIVE

The **derivative** of $f(x)$ **with respect to x** is the function $f'(x)$ and is defined as

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) \quad f(x)}{h} \qquad \text{or} \qquad f'(x) = \lim_{x \to a} \frac{f(x) \quad f(a)}{x \quad a}$$



Let c, b, and n be constants and f, g, functions of x.

$\frac{d}{dx}(c) = 0$	$\frac{d}{dx}(x) = 1$
$\frac{d}{dx}(c \cdot f) = c \cdot f'$	$\frac{d}{dx}(ax^n) = n \cdot ax^{n \quad 1}$ (Power Rule)
$\frac{d}{dx}(f \pm g) = f' \pm g'$	$\frac{d}{dx}(c^x) = c^x \cdot \ln(c)$
$\frac{d}{dx}(f \cdot g) = f \cdot g' + g \cdot f'$ (Product Rule)	$\frac{d}{dx}(\frac{f}{g}) = \frac{g \cdot f' \quad f \cdot g'}{g^2}$ (Quotient Rule)
$\frac{d}{dx}(\ln(f)) = \frac{1}{f} \cdot f'$ for $f \neq 0$	$\frac{d}{dx}(\log_c(f)) = \frac{1}{f \cdot \ln(c)} \cdot f'$ for $f \neq 0$
$\frac{d}{dx}(e^f) = e^f \cdot f'$	$\frac{d}{dx}(f(g)) = f'(g) \cdot g'$ (Chain Rule)

TRIGONOMETRIC DERIVATIVES

$\frac{d}{dx} \sin(x) = \cos(x) \cdot x'$	$\frac{d}{dx} \cos(x) = \quad \sin(x) \cdot x'$
$\frac{d}{dx} \tan(x) = \sec^2(x) \cdot x'$	$\frac{d}{dx} \csc(x) = \quad \csc(x)\cot(x) \cdot x'$
$\frac{d}{dx} \sec(x) = \sec(x)\tan(x) \cdot x'$	$\frac{d}{dx} \cot(x) = \quad \csc^2(x) \cdot x'$
$\frac{d}{dx} \sin^{\quad 1}(x) = \frac{1}{\sqrt{1 \quad x^2}} \cdot x'$	$\frac{d}{dx} \cos^{\quad 1}(x) = \quad \frac{1}{\sqrt{1 \quad x^2}} \cdot x'$
$\frac{d}{dx} \tan^{\quad 1}(x) = \frac{1}{1+x^2} \cdot x'$	*Note* : Here x' was shown to demonstrate the chain rule. In these examples, $x' = 1$, as it is the derivative of x.

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CALCULUS

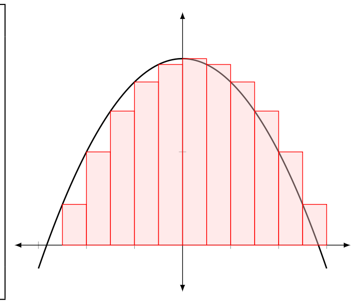
INTEGRAL RULES

DEFINITION OF THE DEFINITE INTEGRAL

If f is integrable on [a,b], then the **integral** of $f(x)$ **with respect to x** is the function $F(x)$ and is defined as

$$F(x) = \int_a^b f(x) \cdot dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \Delta x$$

where $\Delta x = \frac{b \quad a}{n}$ and $x_i = a + i\Delta x$.



PERTINENT SUMS

$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$	$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$	$\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$

FUNDAMENTAL THEOREM OF CALCULUS

If $f(x)$ is continuous on [a,b], then the **integral** of $f(x)$ **with respect to x from a to b** is

$$\int_a^b f(x) \cdot dx = F(b) \quad F(a)$$

where $F(x)$ is an antiderivative of $f(x)$.

Let c, b, and n be constants and f,g, functions of x.

$\int c \cdot f(x) \cdot dx = c \int f(x) \cdot dx$	$\int [f(x) \pm g(x)] \cdot dx = \int f(x) \cdot dx \pm \int g(x) \cdot dx$
$\int dx = x + C$	$\int x^n \cdot dx = \frac{x^{n+1}}{n+1} + C$, for $n \neq \quad 1$
$\int \frac{dx}{x} = ln\lvert x\rvert + C$	$\int e^x \cdot dx = e^x + C$
$\int_a^a f(x) \cdot dx = 0$	$\int_a^b f(x) \cdot dx = \quad \int_b^a f(x) \cdot dx$
$\int \frac{1}{\sqrt{1 \quad x^2}} \cdot dx = \arcsin(x) + C$	$\int \quad \frac{1}{\sqrt{1 \quad x^2}} \cdot dx = \arccos(x) + C$