



# LIMITS

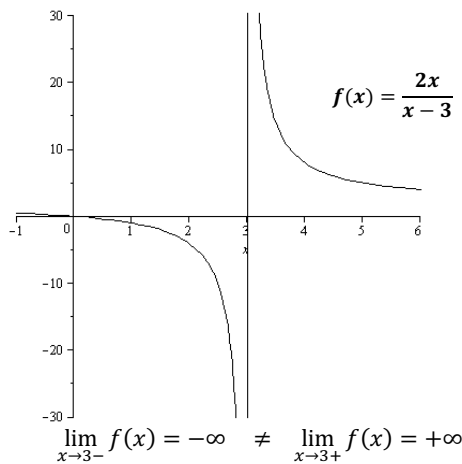
# CALCULUS

**DEFINITION:** We say that the limit of  $f(x)$  equals  $L$  as  $x$  approaches  $a$ , written as:

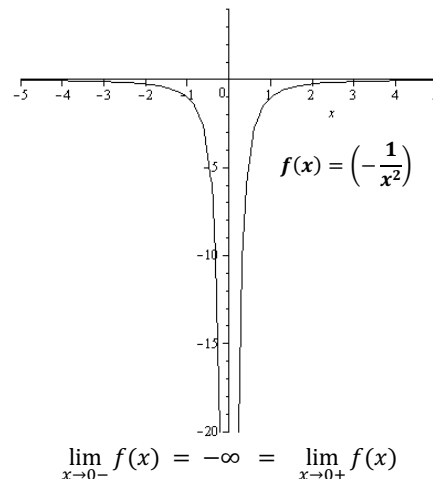
$$\lim_{x \rightarrow a} f(x) = L$$

If we can make the values of  $f(x)$  arbitrarily close to  $L$  by taking  $x$  to be sufficiently close to  $a$  (on either side of  $a$ ) but not equal to  $a$ .

**THEOREM:**  $\lim_{x \rightarrow a} f(x) = L$  if and only if  $\lim_{x \rightarrow a^+} f(x) = L$  and  $\lim_{x \rightarrow a^-} f(x) = L$



The Limit Does Not Exist



Thus,  $\lim_{x \rightarrow 0} f(x) = -\infty$

**Limit Laws:** Suppose  $c$  is a constant and that  $\lim_{x \rightarrow a} f(x)$  and  $\lim_{x \rightarrow a} g(x)$  both exist

$$\lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x)$$

$$\lim_{x \rightarrow a} [f(x)]^n = [\lim_{x \rightarrow a} f(x)]^n$$

$$\lim_{x \rightarrow a} cf(x) = c \lim_{x \rightarrow a} f(x)$$

$$\lim_{x \rightarrow a} x^n = a^n$$

$$\lim_{x \rightarrow a} f(x)g(x) = \lim_{x \rightarrow a} f(x) \lim_{x \rightarrow a} g(x)$$

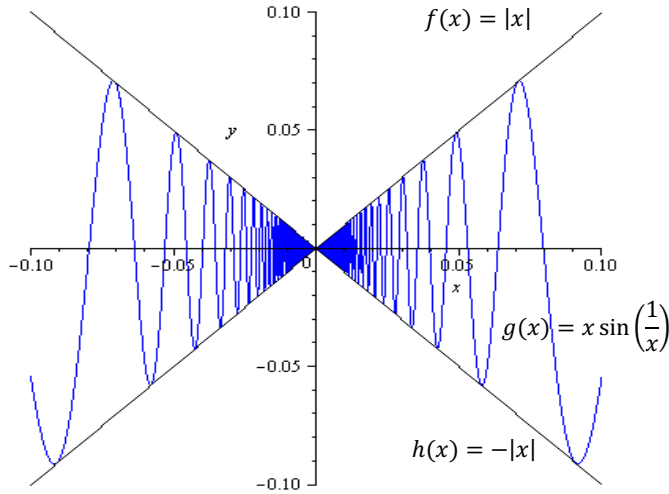
$$\lim_{x \rightarrow a} c = c$$

$$\lim_{x \rightarrow a} \left(\frac{f(x)}{g(x)}\right) = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} \text{ for } g(x) \neq 0$$

**SQUEEZE THEOREM:** If  $f(x) \leq g(x) \leq h(x)$ , when  $x$  is near  $a$ , and  $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} h(x) = L$ , then  $\lim_{x \rightarrow a} g(x) = L$



**Example 1(Using Squeeze Theorem):**



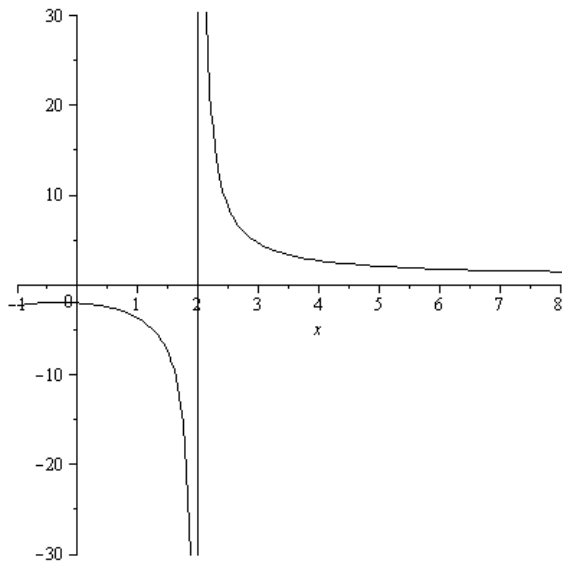
**Observation for example 2:**

Find  $\lim_{x \rightarrow a^+} f(x)$  and  $\lim_{x \rightarrow a^-} f(x)$

By observing the behavior of  $x$  as it approaches  $a$  from both the left and right hand sides, we can calculate the limit.

$$\lim_{x \rightarrow 2^+} \frac{x^2 + 2x + 8}{x^2 - 4} = \infty \quad \lim_{x \rightarrow 2^-} \frac{x^2 + 2x + 8}{x^2 - 4} = -\infty$$

**Example 2:**



$x$	$f(x)$	$x$	$f(x)$
1	-3.66667	3	4.6
1.8	-17.4211	2.5	10.7778
1.9	-38.9673	2.1	40.5122
1.99	-399.499	2.01	400.501
1.9999	40008.5	2.0001	39991.5

$\lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x)$	$\lim_{x \rightarrow a} x = a$
$\lim_{x \rightarrow a} f(x)g(x) = \lim_{x \rightarrow a} f(x)\lim_{x \rightarrow a} g(x)$	$\lim_{x \rightarrow \infty} \left(\frac{f(x)}{g(x)}\right) = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$ for $g(x) \neq 0$
$\lim_{x \rightarrow a} [f(x)]^n = [\lim_{x \rightarrow a} f(x)]^n$	$\lim_{x \rightarrow a} x^n = a^n$
$\lim_{x \rightarrow a} cf(x) = c \lim_{x \rightarrow a} f(x)$	$\lim_{x \rightarrow a} c = c$