California State University SAN MARCOS

Calculus

Partition, Critical & Inflection Numbers

What Are They?

Partition Number	Where $f(x) = 0$ or where $f(x)$ is undefined	x values where f **might** change sign
Critical Number	Where $f'(x) = 0$ or where $f'(x)$ is undefined	x values where f' **might** change sign
Inflection Number	Where $f''(x) = 0$ or where $f''(x)$ is undefined	x values where f'' **might** change sign

- **Partition Number** - Determines open intervals where $f(x)$ does not change sign
- **Critical Number** - Really just a partition number for $f'(x)$, but in the domain of f
- **Inflection Number** - Really just a partition number for $f''(x)$, but in the domain of f

What Can We Use These For?

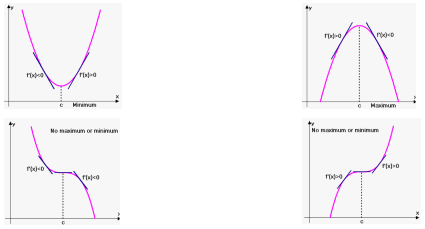
Partition Number	→ Vertical Asymptotes → x-intercepts
Critical Number	→ Minimums and Maximums → Intervals where $f(x)$ is increasing or decreasing
Inflection Number	→ Minimums and Maximums → Intervals where $f(x)$ is concave up or concave down

How Do We Use Them?

Partition Numbers	**Critical Numbers**	**Inflection Numbers**
1. $f(x) = 0$ and solve for x • These are the x-intercepts **2.** Find any domain restrictions for $f(x)$	**1.** Find $f'(x)$ **2.** Set $f'(x) = 0$ and solve for x **3.** Find any domain restrictions for $f'(x)$ **4.** Make sure all numbers found in 2. and 3. are in the domain of f • These are the critical numbers for f **5.** Test values in $f'(x)$ on either side of each critical number. **6.** Use the First Derivative Test table below to analyze the results	**1.** Find $f''(x)$ **2.** Set $f''(x) = 0$ and solve for x **3.** Find any domain restrictions for $f''(x)$ **4.** Make sure all numbers found in 2. and 3. are in the domain of f • These are the inflection numbers for f **5.** Test values in $f''(x)$ on either side of each inflection number. **6.** Use the Second Derivative Test table below to analyze the results

First Derivative Test

$f(x)$ **left of c**	$f(x)$ **right of c**	$f(c)$
Decreasing	Increasing	Local minimum at c
Increasing	Decreasing	Local maximum at c
Decreasing	Decreasing	Not an extremum
Increasing	Increasing	Not an extremum



Second Derivative Test

$f'(c)$	$f''(c)$	**graph of** f is	$f(c)$ is
0	+	Concave Up	Local Minimum
0	-	Concave Down	Local Maximum
0	0	?	Test Fails