Guide To The K-Map (Karnaugh Map)

In many digital circuits and practical problems we need to find expression with minimum variables. We can minimize Boolean expressions of 2, 3, or 4 variables very easily using the K-map without using any Boolean algebra theorems. The K-map can take two forms Sum of Product (SOP) and Product of Sum (POS) according to the needs of the problem. The K-map is table-like representation but it gives more information than TRUTH TABLE. We fill the grid of K-map with 0's and 1's then solve it by making groups.

Steps to solve expression using the K-map

1. Select K-map according to the number of variables.
2. Identify minterms or maxterms as given in the problem.
3. For SOP put 1's in blocks of K-map respective to the minterms (0's elsewhere).
4. For POS put 0's in blocks of K-map respective to the maxterms(1's elsewhere).
5. Make rectangular groups containing total terms in power of two like 2,4,8 ..(except 1) and try to cover as many elements as you can in one group.
6. From the groups made in step 5 find the product terms and sum them up for SOP form.

The K-map Fill Order

2 - Variable Map

A\B	0	1
0	0	1
1	2	3

3- Variable Map

A\BC	00	01	11	10
0	0	1	3	2
1	4	5	7	6

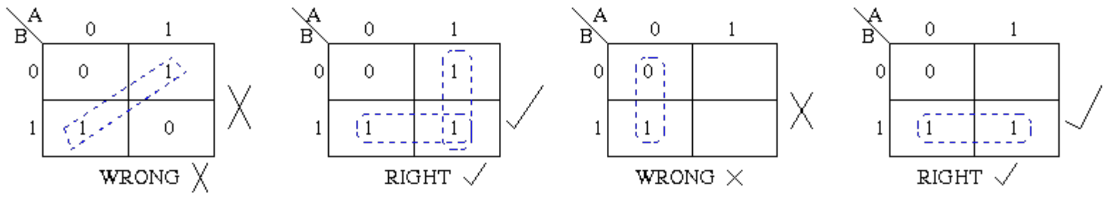
4 - Variable Map

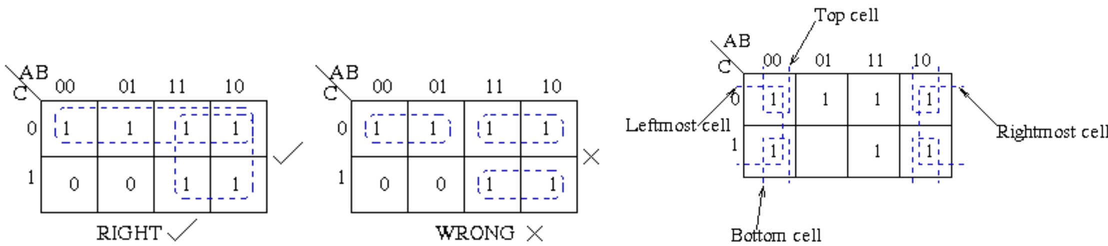
AB\CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

Grouping Rules

The Karnaugh map uses the following rules for the simplification of expressions by grouping together **adjacent cells containing ones**

1. No zeros allowed.
2. No diagonals.
3. Only power of 2 number of cells in each group.
4. Groups should be as large as possible.
5. Everyone must be in at least one group.
6. Overlapping allowed.
7. Wrap around is allowed.
8. Get the fewest number of groups possible.





We perform the **Sum of minterm** also known as **Sum of products** (SOP).

- The **minterm** for each combination of the variables that produce a **1** in the function and then taking the **OR** of all those terms.

We perform the **Product of Maxterm** also known as **Product of sum** (POS).

- The **maxterm** for each combination of the variables that produce a **0** in the function and then taking the **AND** of all those terms.

Truth table representing minterm and maxterm

				Minterms		Maxterms
X	**Y**	**Z**		**Product Terms**		**Sum Terms**
0	**0**	**0**		$m_0= \underline{X} \cdot \underline{Y} \cdot \underline{Z}$ = min($\underline{X}, \underline{Y}, \underline{Z}$)		$M_0= X + Y + Y$ = max(X, Y, Z)
0	**0**	**1**		$m_1= \underline{X} \cdot \underline{Y} \cdot Z$ = min($\underline{X}, \underline{Y}, Z$)		$M_1= X + Y + \underline{Z}$ = max($X, Y, \underline{Z}$)
0	**1**	**0**		$m_2= \underline{X} \cdot Y \cdot \underline{Z}$ = min($\underline{X}, Y, \underline{Z}$)		$M_2= X + \underline{Y} + Z$ = max($X, \underline{Y}, Z$)
0	**1**	**1**		$m_3= \underline{X} \cdot Y \cdot Z$ = min($\underline{X}, Y, Z$)		$M_3= X + \underline{Y} + \underline{Z}$ = max($X, \underline{Y}, \underline{Z}$)
1	**0**	**0**		$m_4= X \cdot \underline{Y} \cdot \underline{Z}$ = min($X, \underline{Y}, \underline{Z}$)		$M_4= \underline{X} + Y + Z$ = max($\underline{X}, Y, Z$)
1	**0**	**1**		$m_5= X \cdot \underline{Y} \cdot Z$ = min($X, \underline{Y}, Z$)		$M_5= \underline{X} + Y + \underline{Z}$ = max($\underline{X}, Y, \underline{Z}$)
1	**1**	**0**		$m_6= X \cdot Y \cdot \underline{Z}$ = min($X, Y, \underline{Z}$)		$M_6= \underline{X} + \underline{Y} + Z$ = max($\underline{X}, \underline{Y}, Z$)
1	**1**	**1**		$m_7= X \cdot Y \cdot Z$ = min(X, Y, Z)		$M_7= \underline{X} + \underline{Y} + \underline{Z}$ = max($\underline{X}, \underline{Y}, \underline{Z}$)

From the table above, it is clear that minterm is expressed in product format and maxterm is expressed in sum format.