CSUSM
STEM Success Center

Difference Quotient Handout

Definition: The difference quotient is a formula that is used in later calculus/math courses. Its formula is:

$$\frac{f(x+h)-f(x)}{h}$$

Procedure: To solve the difference quotient,

1) Determine $f(x)$ from what is given from the problem and determine $f(x+h)$ by replacing all x in $f(x)$ with $x+h$.
2) Simplify the numerator $f(x+h)-f(x)$ (Don't forget to distribute the negative into $f(x)$!)
3) Factor out an h from the numerator and cancel the h in the denominator for your answer.

Examples:

Determine $\frac{f(x+h)-f(x)}{h}$ if $f(x)=x+2$.

1) Determine $f(x)$ from what is given from the problem and determine $f(x+h)$ by replacing all x in $f(x)$ with $x+h$.

Our $f(x) = x+2$ from what we are given. To determine $f(x+h)$, we replace the x in $x+2$ with $x+h$, giving us $(x+h)+2$. Plugging into $\frac{f(x+h)-f(x)}{h}$ gives us $\frac{(x+h)+2-(x+2)}{h}$.

2) Simplify the numerator $f(x+h)-f(x)$ (Don't forget to distribute the negative into $f(x)$!)

We will simplify the numerator, namely $(x+h)+2-(x+2)$. We can drop the parentheses around $x+h$ since it is not being distributed by anything. Distributing the negative to $x+2$ gives us $x-2$. In summary, we have:

$$(x+h)+2-(x+2)$$

$$x+h+2-(x+2)$$

$$x+h+2-x-2$$

$$\cancel{x}+h+\cancel{2}-\cancel{x}-\cancel{2}$$

Cancelling out the like terms leave us with h in the numerator.

3) Factor out an h from the numerator and cancel the h in the denominator for your answer.

Overall, we are left with $\frac{h}{h}$ since we originally had an h in the denominator as well. We can divide h by h, leaving us with 1 as our final answer. In other words, $\frac{h}{h} = 1$.

If $f(x) = x^2 - 4$, find $\frac{f(x+h)-f(x)}{h}$.

1) Determine $f(x)$ from what is given from the problem and determine $f(x+h)$ by replacing all x in $f(x)$ with $x+h$.

Our $f(x) = x^2 - 4$ from what we are given, so our $f(x+h) = (x+h)^2 - 4$ from replacing the x in $x^2 - 4$ with $x+h$. Plugging into $\frac{f(x+h)-f(x)}{h}$ gives us $\frac{(x+h)^2-4-(x^2-4)}{h}$.

2) Simplify the numerator $f(x+h) - f(x)$ (Don't forget to distribute the negative into $f(x)$!)

We will simplify the numerator, namely $(x+h)^2 - 4 - (x^2 - 4)$. First, we will need to expand $(x+h)^2$ and use the FOIL method:

$$(x+h)^2 = (x+h)(x+h) = x^2 + 2hx + h^2$$

This gives us $x^2 + 2hx + h^2 - 4 - (x^2 - 4)$. Now, we will need to distribute the negative into $x^2 - 4$:

$$x^2 + 2hx + h^2 - 4 - x^2 + 4$$

Next, we cancel out the like terms:

$$\cancel{x^2} + 2hx + h^2 - 4 - \cancel{x^2} + 4$$

$$2hx + h^2$$

3) Factor out an h from the numerator and cancel the h in the denominator for your answer.

Overall, we are left with $\frac{2hx+h^2}{h}$ since we originally had an h in the denominator as well. We can factor our an h from $2hx + h^2$, giving us $\frac{h(2x+h)}{h}$. Finally, we will cancel out h, leaving us with our final answer of $(2x+h)$.