California State University
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Math 270 & 350

1 Logical Statements

Negation (Not):

If p is a statement variable, the negation of p is "not p" and is denoted $\sim p$. It has opposite truth value from p: if p is true, $\sim p$ is false; if p is false, $\sim p$ is true.

Conjunction (And):

If p and q are statement variables, the conjunction of p and q is "p and q", denoted $p \wedge q$. It is true when, and only when, both p and q are true. If either p or q is false, or if both false, $p \wedge q$ is false.

Disjunction (Or):

If p and q are statement variables, the disjunction of p and q is "p or q", denoted $p \vee q$. It is true when either p is true, or q is true, or both p and q are true; it is false only when both p and q are false.

Logical Equivalence:

Two statement forms are called logically equivalent $\iff$ they have identical truth values for each possible substitution of statements for their statement variables. The logical equivalence of statement forms P and Q is denoted by writing $P \equiv Q$. Two statements are called logically equivalent $\iff$, they have logically equivalent forms when identical component statement variables are used to replace identical component statements.

De Morgan's Laws:

The negation of an and statement is logically equivalent to the or statement in which each component is negated.

$\sim (p \wedge q) \equiv \sim p \vee \sim q$

The negation of an or statement is logically equivalent to the and statement in which each component is negated.

$\sim (p \vee q) \equiv \sim p \wedge \sim q$

Contradiction Statements:

A contradiction is a statement form that is always false regardless of the truth values of the individual statements substituted for its statement variables. A statement whose form is a contradiction is a contradictory statement.

Conditional Statements:

If p and q are statement variables, the conditional of q by p is "If p then q" or "p implies q" and is denoted $p \implies q$. It is false when p is true and q is false; otherwise, it is true. We call p the hypothesis (or antecedent) of the conditional and q the conclusion (or consequent).

Negation of Conditional Statements:

The negation of "if p then q" is logically equivalent to "p and not q".

Contrapositive of Conditional Statements:

The contrapositive of a conditional statement of the form "If p then q" is "If $\sim q$ then $\sim p$". Symbolically of $p \implies q$ is $\sim q \implies \sim p$.

Converse & Inverse of Conditional Statements:

Suppose a conditional statement of the form "If p then q" is given.
1. The converse is "If q then p".
2. The inverse is "If $\sim p$ then $\sim q$".

Symbolically:
1. The converse of $p \implies q$ is $q \implies p$.

Note that: $p \implies q$ is not the same as $q \implies p$.

2. The inverse of $p \implies q$ is $\sim p \implies \sim q$

Biconditional Statements:

Given statement variables p and q, the biconditional of p and q is "p if, and only if, q" and is denoted $p \iff q$. It is true if both p and q have the same truth values and is false if p and q have opposite truth values. The words if and only if are sometimes abbreviated iff.

2 Order of Operations for Logical Operators

1. Evaluate Negations first ($\sim$).

2. Evaluate $\wedge$ and $\vee$ second. When both are present, parentheses may be needed.

3. Evaluate $\implies$ and $\iff$ third. When both are present, parentheses may be needed.

p	$\sim p$
T	F
F	T

Table 1: Negation Truth Table (Not)

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Table 2: Conjunction Truth Table (And)

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Table 3: Disconjunction Truth Table (Or)

p	q	$p \implies q$
T	T	T
T	F	F
F	T	T
F	F	T

Table 4: Conditional Truth Table (Implies)

p	q	$p \iff q$
T	T	T
T	F	F
F	T	F
F	F	T

Table 5: Biconditional Truth Table (Iff)