Math 132 Local Extrema Worksheet

For all problems, find the critical values, determine the local extrema, and state whether the local extrema is a minimum or maximum.

1) $y = 2x^2 - 8x + 4$

2) $y = 4x^2 + 15x + 7$

3) $y = \frac{1}{3}x^3 + 3x^2 + 8x + 3$

4) $y = 2x^3 + 7x^2 + 4x + 5$

Solutions:

1) Find the derivative and set it equal to 0 for the critical values:

$$y' = 4x - 8 = 0$$

$$x = 2$$

2 is your critical value, so use a number less than 2 and a number greater than 2 as test points and plug those test points into y'. We will use 0 and 3:

0	2	3
	Critical Value	

We plug 0 and 3 into y':

$$4(0) - 8 = -8$$

$$4(3) - 8 = 4$$

When $x = 0$, we get a negative number. When $x = 3$, we get a positive number. It turns out that any number you plug in that is less than 2 will also be negative, and any number you plug in that is greater than 2 will also be positive. Since the equation we are plugging into represents the derivative, the x-values before 2 will cause the graph of y to decrease, and the x-values after 2 will cause the graph of y to increase. We will update our table to reflect this:

0	2	3
-	Critical Value	+

We can now picture what will happen with this information. Since the graph of y decreases up to 2 and then increases after 2, we can imagine that the point at $x = 2$ will be a local minimum. Lastly, we will plug 2 into y to find the local minimum:

$$y = 2(2)^2 - 8(2) + 4 = -4$$

Therefore, we will have a local minimum at $(2, -4)$.

2) Find the derivative and set it equal to 0 for the critical values:

$$y' = 8x + 15 = 0$$

$$x = -\frac{15}{8}$$

$-\frac{15}{8}$ is your critical value, so use a number less than $-\frac{15}{8}$ and a number greater than $-\frac{15}{8}$ as test points and plug those test points into y'. We will use -2 and 0:

-2	-15/8	0
	Critical Value	

We plug -2 and 0 into y':

$$8(-2) + 15 = -1$$

$$8(0) + 15 = 15$$

When $x = -2$, we get a negative number. When $x = 0$, we get a positive number. It turns out that any number you plug in that is less than $-\frac{15}{8}$ will also be negative, and any number you plug in that is greater than $-\frac{15}{8}$ will also be positive. Since the equation we are plugging into represents the derivative, the x-values before $-\frac{15}{8}$ will cause the graph of y to decrease, and the x-values after $-\frac{15}{8}$ will cause the graph of y to increase. We will update our table to reflect this:

-2	-15/8	0
-	Critical Value	+

We can now picture what will happen with this information. Since the graph of y decreases up to $-\frac{15}{8}$ and then increases after $-\frac{15}{8}$, we can imagine that the point at $x = -\frac{15}{8}$ will be a local minimum. Lastly, we will plug -15/8 into y to find the local minimum:

$$4\left(-\frac{15}{8}\right)^2 + 15\left(-\frac{15}{8}\right) + 7 = -\frac{113}{16}$$

Therefore, we will have a local minimum at $(-\frac{15}{8}, -\frac{113}{6})$

3) Find the derivative and set it equal to 0 for the critical values:

$$y' = x^2 + 6x + 8 = 0$$

$$(x + 4)(x + 2) = 0$$

$$x = -4, -2$$

-4 and -2 are your critical values, so use a number less than -4, a number between -4 and -2, and a number greater than -2 as test points and plug those test points into y'. We will use -5, -3, and 0:

-5	-4	-3	-2	0
	Critical Value		Critical Value	

We plug -5, -3, and 0 into y':

$$y' = (-5+4)(-5+2) = 3$$

$$y' = (-3+4)(-3+2) = -1$$

$$y' = (0+4)(0+2) = 8$$

When $x = -5$, we get a positive number. When $x = -3$, we get a negative number. When $x = 0$, we get another positive. It turns out that any number you plug in that is less than -4 will also be positive, any number you plug in that is between than -4 and -2 will be negative, and any number you plug in that is greater than -2 will be positive. Since the equation we are plugging into represents the derivative, the x-values before -4 will cause the graph of y to increase, the x-value between -4 and -2 will cause the graph of y to decrease, and the x-values after -2 will cause the graph of y to decrease. We will update our table to reflect this:

-5	-4	-3	-2	0
+	Critical Value	-	Critical Value	+

We can now picture what will happen with this information. Since the graph of y increases up to $x = -4$ and then decreases after $x = -4$, we can imagine that the point at $x = -4$ will be a local maximum. Then, the graph of y continues to decrease up to $x = -2$ and then increases again after. So, we can imagine the point at $x = -2$ to be a local minimum. Lastly, we will plug -2 into y to find the local minimum, and -4 into y to find the local maximum:

$$y = \frac{1}{3}(-2)^3 + 3(-2)^2 + 8(-2) + 3 = -\frac{11}{3}$$

$$y = \frac{1}{3}(-4)^3 + 3(-4)^2 + 8(-4) + 3 = -\frac{7}{3}$$

So, we have a local minimum at $(-2, -\frac{11}{3})$ and a local maximum at $(-4, -\frac{7}{3})$.

4) Find the derivative and set it equal to 0 for the critical values:

$$y' = 6x^2 + 14x + 4 = 0$$

We will factor using the A-C Method. We will label 6 as a, 14 as b, and 4 as c. We know $a * c = 6 * 4 = 24$ and $b = 14$.

We need two numbers that multiply to 10 and add to 14. These two numbers are 2 and 12. So, we will rearrange our equation accordingly:

$$y' = 6x^2 + 14x + 4 = 0$$

$$y' = 6x^2 + 12x + 2x + 4 = 0$$

$$y' = 6x(x+2) + 2(x+2) = 0$$

$$y' = (6x+2)(x+2) = 0$$

Now we can find our critical values by setting each factor equal to zero:

$$6x + 2 = 0$$

$$x = -\frac{1}{3}$$

$$x + 2 = 0$$

$$x = -2$$

So, our critical values will be at $x = -\frac{1}{3}$ and $x = -2$, so use a number less than -2, a number between -2 and -1/3, and a number greater than -1/3 as test points and plug those test points into y'. We will use -3, -1, and 0:

$$y' = (6(-3)+2)\big((-3)+2\big) = 16$$

$$y' = (6(-1)+2)\big((-1)+2\big) = -4$$

$$y' = (6(0)+2)\big((0)+2\big) = 4$$

When $x = -3$, we get a positive number. When $x = -1$, we get a negative number. When $x = 0$, we get another positive. It turns out that any number you plug in that is less than -2 will also be positive, any number you plug in that is between than -2 and $-\frac{1}{3}$ will be negative, and any number you plug in that is greater than $-\frac{1}{3}$ will be positive. Since the equation we are plugging into represents the derivative, the x-values before -2 will cause the graph of y to increase, the x-values between -2 and $-\frac{1}{3}$ will cause the graph of y to decrease, and the x-values after $-\frac{1}{3}$ will cause the graph of y to increase. We will create a table to reflect this:

-3	-2	-1	-1/3	0
+	Critical Value	-	Critical Value	+

We can now picture what will happen with this information. Since the graph of y increases up to $x = -2$ and then decreases after $x = -2$, we can imagine that the point at $x = -2$ will be a local maximum. Then, the graph of y continues to decrease up to $x = -\frac{1}{3}$ and then increases again after. So, we can imagine the point at $x = -\frac{1}{3}$ to be a local minimum. Lastly, we will plug -1/3 into y to find the local minimum, and -2 into y to find the local maximum:

$$y = 2\left(-\frac{1}{3}\right)^3 + 7\left(-\frac{1}{3}\right)^2 + 4\left(-\frac{1}{3}\right) + 5 = \frac{118}{27}$$

$$y = 2(-2)^3 + 7(-2)^2 + 4(-2) + 5 = 9$$

So, we have a local minimum at $(-\frac{1}{3}, \frac{118}{27})$ and a local maximum at $(-2, 9)$.