Math 132 Maximizing Revenue/Profit Worksheet

1) A bicycle company sells x bicycles at a rate of $\$p$ per bicycle. The price-demand equation for these bicycles is $p = 200 - .004x$. What is the maximum revenue? What price should the bicycle company charge for the bicycles to maximize revenue?

2) The total annual cost of manufacturing x bicycles for the bicycle company in problem 1 is $C(x) = 1000 + 3x$. What is the company's maximum profit? What should the company charge for each bicycle, and how many bicycles should be produced?

3) The government decides to tax the company in the previous example $20 for each bike produced. Considering this additional cost, how many markers should the company manufacture annually to maximize this profit? What is the maximum profit? How much should the company charge for the bicycles to realize the maximum profit?

4) When the Padres price their games at about $50 per person, 30000 people will attend the game. The Padres estimate that for each $2 reduction in price, 100 more people will attend the game. How much should the Padres charge for the game to maximize the revenue? What is the maximum revenue?

Solutions:

1) We know our price equation is $p = 200 - .004x$. We can derive the revenue equation by multiplying p by x:

$$xp = x(200 - .004x) = 200x - .004x^2 = R(x)$$

Now that we have $R(x)$, we now find the marginal revenue and set it equal to 0 for our critical value(s):

$$R'(x) = 200 - .008x = 0$$

$$x = 25000$$

To double check if the critical value/demand holds the maximum revenue, we will find the second derivative of $R(x)$:

$$R''(x) = -.008$$

Since the second derivative turns out to be negative, the revenue of our demand of \$25000 will indeed be the maximum revenue. So lastly, we will find our maximum revenue by plugging in our demand of \$25000 into $R(x)$:

$$200(25000) - .004(25000)^2 = \$2500000$$

So, we will have a maximum revenue \$2500000 when 25000 bikes are produced. To find the price we should charge for the maximum revenue, we will plug in 25000 into the original price equation we were given:

$$200 + .004(25000) = \$300$$

2) We will derive the profit function $P(x)$ by subtracting the cost function from the revenue function:

$$P(x) = R(x) - C(x)$$

$$P(x) = 200x - .004x^2 - (1000 + 3x)$$

$$P(x) = 200x - .004x^2 - 1000 - 3x$$

$$P(x) = -.004x^2 + 197x - 1000$$

We then find the critical value/demand by taking the derivative of $P(x)$ setting it equal to 0:

$$P'(x) = -.008x + 197 = 0$$

$$x = 24625$$

Next, we take the second derivative of $P(x)$ to confirm we will have a maximum profit.

$$P''(x) = -.008$$

Once again, our second derivative is negative, confirming we will have a maximum profit. To find the maximum profit we will plug in our demand of 24625 into $P(x)$:

$$P(24625) = -.004(24625)^2 + 197(24625) - 1000 = \$2424562.5$$

So, we will have a maximum profit of \$2424562.5 when 24625 bikes are sold. To find the price we should charge for our maximum profit, we plug 24625 into the price function:

$$p = 200 - .004(24625) = \$101.5$$

3) One piece of additional information here is the extra money being charged from tax. We know that $C(x) = original\ cost + tax$. We also know that for each bike manufactured, it is an extra \$20 from the government, so $tax = 20x$. Using our original cost (already given in the previous problem) and our tax, we have:

$$C(x) = 1000 + 3x + 20x = 1000 + 23x$$

We will then find $P(x)$ by doing $R(x) - C(x)$:

$$P(x) = 200x - .004x^2 - (1000 + 23x)$$

$$P(x) = 200x - .004x^2 - 1000 - 23x$$

$$P(x) = 177x - .004x^2 - 1000$$

We will now find marginal profit by taking the derivative, and then find the demand that leads to the maximum revenue by setting the marginal profit equal to 0:

$$P'(x) = 177 - .008x = 0$$

$$x = 22125$$

Our second derivative will be negative, confirming the maximum profit. We now plug in 22125 into the profit function to get our maximum profit:

$$P(22125) = 177(22125) - .004(22125)^2 - 1000 = \$1957062.5$$

Lastly, we plug in our supply demand into the price equation to see how much we should charge to realize the maximum profit:

$$p = 200 - .004(22125) = \$111.5$$

4) We know two things: when the Padres make their tickets \$50, 30000 people attend the game. They also estimate that if they reduce the price by \$2, 1000 more people will attend the game. We know that $R(x) = price\ per\ customer * number\ of\ customers$:

$$R(x) = (\$50 - 2x)(30000 + 1000x)$$

$$R(x) = 1500000 - 10000x - 2000x^2$$

Since price cannot be negative, we must also remember that $50 - 2x \geq 0$, so $x \leq 25$ or in other words $25 \geq x$.

We now find the marginal revenue and set it equal to 0 for the critical value associated with the maximum revenue:

$$R'(x) = 10000 - 4000x = 0$$

$$x = 2.5$$

The second derivative of $R(x)$ will be negative, assuring a maximum revenue. So, we will plug 2.5 into $R(x)$ for our maximum revenue:

$$R(x) = 1500000 - 10000(2.5) - 2000(2.5)^2 = \$1462500$$

So, the Padres will realize a maximum revenue of \$1462500. Lastly, we will find the price they should charge by plugging in our critical value into the price equation:

$$p = 50 - 2(2.5) = \$45$$