1. Rectangular form to polar form

Change \( x^2 + y^2 - 2y = 0 \) to polar form

Solution:

Use: \( r^2 = x^2 + y^2 \)

\[
x^2 + y^2 - 2y = 0 \quad [\text{Replace } (x^2 + y^2) \text{ with } r^2]
\]

\[
r^2 - 2y = 0
\]

\[
r^2 - 2(r \sin(\theta)) \quad [\text{replace } y \text{ with } r \sin(\theta)]
\]

\[
r(r - 2\sin(\theta)) = 0 \quad [\text{factor out } r]
\]

We get \( r = 0 \), or \( r - 2\sin(\theta) = 0 \)

The graph of \( r = 0 \) is the pole. (It represents one point only)
The pole is included in the graph of \( r - 2\sin(\theta) = 0 \)
We can discard \( r = 0 \) and just keep

\[
r - 2\sin(\theta) = 0
\]

\[
r = 2\sin(\theta) \quad [\text{The polar form of } x^2 + y^2 - 2y = 0]
\]

2. Polar to Rectangular

Change \( r = -3 \cos(\theta) \) to rectangular form

Solution:

Use: \( r^2 = x^2 + y^2 \)

\[
r = -3 \cos(\theta) \quad [\text{Multiply by } r \text{ to get } r^2]
\]

\[
r^2 = -3r \cos(\theta)
\]

\[
x^2 + y^2 = -3r \cos(\theta) \quad [\text{Use } x = r \cos(\theta)]
\]
Conversion: Rectangular to Polar/ Polar to Rectangular

\[ x^2 + y^2 + 3x = 0 \]  
[Rectangular form]

\[(x^2 + 3x) + y^2 = 0 \]  
reorganize in \(x^2 + y^2 = r^2\)

\[(x^2 + 3x + \frac{9}{4}) + y^2 = 0 + \frac{9}{4}\]  
[complete the square]

\[(x + \frac{3}{2})^2 + y^2 = \frac{9}{4}\]  
[rectangular form]

Ex: given point = 4 at 30degree.

Convert to rectangular:

\[ y = r \sin(\theta), x = r \cos(\theta) \]  
so \([x, y] = [4\cos(30), 4\sin(30)] = [2\sqrt{3}, 2]\]

In general:

Use: \(r^2 = x^2 + y^2\)

and either  
\[y = r \sin(\theta)\] (when \(y\) is the term used in the original equation)

or  
\[x = r \cos(\theta)\] (when \(x\) is the term used in the original equation)

Sometimes it is helpful to multiply the whole equation times \(r\) as a first step, as seen above.

If you end up with \(r = "some value"\), the plot of this is just a circle with \(r\) radius.